Econ 802

Second Midterm Exam

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All questions have equal weight. You may want to work first on the questions where you are most confident.

1. Here are some Chapter 5 questions. In each part, justify your reasoning. You can assume that functions are differentiable and optimal input bundles are interior.

(a) Consider the cost function c(y) where $y > 0$ is a scalar output and input prices are held constant. Prove that if average cost is falling, marginal cost is below it, and if average cost is rising, marginal cost is above it.

(b) Let c(w, y) be the long run cost function and let $c(w, y, x_n)$ be the short run cost function where input n is fixed at x_n. The long run average cost curve is rising at the output $y^* > 0$. Prove that the long run average cost curve cannot pass through the minimum point of the short run average cost curve at y*.

(c) The input prices are $w = (w_1 \ldots w_n) > 0$ and the firm's production function is $y = f(x_1 \ldots x_n)$ with $x \geq 0$. Prove that the long run marginal cost at the output $y > 0$ is equal to the Lagrange multiplier in the cost minimization problem for y.

2. Joe Circle has simple preferences. For each consumption bundle $x = (x_1, x_2) \geq 0$, he computes the distance of x from the origin. If this distance is less than 1.0, he has $u(x) = 0$. If this distance is greater than or equal to 1.0, he has $u(x) = 1$.

(a) Draw a graph showing the consumption bundles for which $u(x) = 0$ and $u(x) = 1$. Do Joe's preferences satisfy (i) weak monotonicity? (ii) strong monotonicity? (iii) convexity? (iv) strict convexity? (v) local non-satiation? Explain briefly.

(b) Joe has the budget constraint $p_1x_1 + p_2x_2 \leq m$ where $0 < p_1 < p_2$ and $m > 0$. Draw one graph where Joe's utility maximization problem has a unique solution, and a second graph where this problem has many solutions. Explain in each case.

(c) If x* solves an expenditure minimization problem, does it also generally solve a utility maximization problem? If x* solves a utility max problem, does it also generally solve an expenditure min problem? Explain graphically.

3. Consider the general Leontief utility function $u = \min \{a_1x_1, a_2x_2 \ldots a_nx_n\}$ where $(x_1, x_2 \ldots x_n) \geq 0$ and the $a_i > 0$ are constants for $i = 1 \ldots n$.

(a) Suppose there are two goods. For some arbitrary consumption bundle $x^* = (x_1^*, x_2^*) > 0$, we have $m = 1$ and we want to identify unique prices $p^* = (p_1^*, p_2^*) > 0$ such that x^* is chosen. What problems will arise? Explain using a graph.

(b) Now consider the n-good case. Without using a graph, give a general argument that enables you to solve for the Hicksian demands $h_i(p, u)$ for $i = 1 \,..\, n$ and the expenditure function $e(p, u)$ where $p > 0$. Justify each step in your logic.

(c) Again consider the n-good case. Without using a graph, give a general argument that enables you to solve for the Marshallian demands $x_i(p, m)$ for $i = 1 \,..\, n$ and the indirect utility function $v(p, m)$ where $p > 0$. Justify each step in your logic.

4. Jane Economist has the direct utility function $u(c, L)$ where $c \geq 0$ is consumption and $L \geq 0$ is leisure. $H + L = T$ where H is work and T is Jane's time endowment. Let $m > 0$ be income, let $p > 0$ be the price of consumption, let w be the wage, and let $0 < \alpha < 1$. Jane's Marshallian demand for leisure is $L(p, w, m) = \alpha m/w$.

(a) Use the Slutsky equation to split $\partial L(p, w, m)/\partial w$ into a substitution effect and an income effect. Interpret the sign of each effect.

(b) Now let $m = wT + r$ where r is non-labor income, so Jane's demand for leisure is $L(p, w, wT + r)$. If $r = 0$, what is true about the relationship between the wage and leisure? Interpret the difference between your results in (a) and your results here.

(c) Compute the optimal consumption (c) as a function of (p, w, r). Do the resulting comparative static effects make economic sense? Explain.

5. Ray Bob maximizes the direct utility function $u(x_1, x_2) = x_1^{1/2} + x_2$ subject to the budget constraint $p_1x_1 + p_2x_2 = m$ where $(p_1, p_2, m) > 0$.

(a) Compute the Marshallian demands $x_1(p, m)$ and $x_2(p, m)$ as well as the indirect utility function $v(p, m)$. Note: don't worry about possible negative values for x_2.

(b) Compute the Hicksian demand $h_1(p, u)$ for good 1. Now suppose the prices p are held constant. Draw a graph in (x_1, x_2) space showing how $x_1(p, m)$ is affected as m changes, and how $h_1(p, u)$ is affected as u changes. Give a verbal explanation.

(c) Suppose we have many consumers $i = 1 \,..\, n$. Their direct utility functions all have the same functional form as before, but with different exponents on x_1. For each consumer the exponent is a positive fraction, but some have 1/2, some have 3/8, some have 5/6, and so on. Is it possible to aggregate across consumers and treat the market demands $X^1(p, M)$ and $X^2(p, M)$ as if they come from one big consumer with the income $M = \sum m_i$? Carefully explain why or why not.